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A SIMPLE METHOD OF OBTAINING SPAN LOAD DISTRIBUTIONS

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SUMMARY

A method based on the principle of superposition is presented for deriving the span load distributions of wings. The procedure involves, as a first step, the determination of a chord distribution that corresponds to an elliptical loading and the given distributions of angle of attack and lift-curve slope. The differences between this fictitious and the given chord distribution define the smaller component loads to be added to the initial elliptical load to produce a new span loading that more nearly satisfies the given problem than did the initial load. The subsequent steps follow a similar procedure; the successive series of additional component loads, however, become rapidly negligible. The method is comparatively quick and simple and should prove useful in problems for which the span load distributions are not otherwise readily obtainable.

INTRODUCTION

In airplane design, problems concerning wing stresses, stalling characteristics, performance at moderate to high angles of attack, and lateral controllability require knowledge of the wing span load distribution. Refined present-day design practice has grown away from the use of arbitrarily assumed curves of span loading and the distributions are derived by methods based on vortex theory. The most successful of such methods have employed Fourier series analyses (references 1 and 2). These methods have been used to make readily available the span load distributions for many representative wing designs (references 3, 4, 5, and 6).

Where a span loading of more than ordinary complexity is involved, a Fourier series method of analysis can become too tedious to be justified. In such a problem, a method of successive approximations (reference 7) employing simple graphical integrations of the fundamental equation for the induced angle at the wing (reference 1) can be resorted to. The labor of deriving the induced-angle distributions associated with the successive approximations can be minimized by graphical dodges or can be practically eliminated by the use, where available, of an integrator designed for the purpose. (See reference 8.) Although the effort entailed by a method of successive approximations may therefore be considerably reduced, these methods suffer the disadvantage of requiring a certain amount of experience and judgment on the part of the engineer. Inasmuch as he seldom has occasion for acquiring such experience or judgment, he is generally reluctant to employ a method of successive approximations when the need arises.

It therefore appears desirable to have some method of deriving span load distributions that is capable, within the limitations of vortex theory, of handling any wing design with a minimum of effort and experience on the part of the user. The purpose of this paper is to present such a method.

SYMBOLS AND DEFINITIONS

It is desirable first to define the symbols and the concepts employed:

b, wing span.

c, wing chord.

y, spanwise distance from wing center line.

α_a, geometric angle of attack in degrees from the angle of zero lift of a section along the span.

c_l, section-lift coefficient, $dL/qcdy$.

L, wing-lift force.

q, dynamic pressure.

a_0, section lift-curve slope in degree measure.

$c_l \frac{c}{b/2}$, nondimensional load intensity at a section along the span.

$\Delta\left(c_l \frac{c}{b/2}\right)$, increment of load intensity at a section along the span.

α_i, induced angle in degrees at a section along the span associated with a given span load distribution and determined from the equation

$$(\alpha_i)_1 = \frac{57.3}{8\pi} \int_{-1}^{1} \frac{d\left(c_l \frac{c}{b/2}\right)}{d\left(\frac{y}{b/2}\right)} \left(\frac{b/2}{y_1 - y}\right) d\left(\frac{y}{b/2}\right)$$

The sign of α_i is taken as positive in a sense opposite to the wing angle of attack.

$\Delta\alpha_i$, increment of induced angle at any section.

It is convenient to write the wing plan-form dimensions as nondimensional ratios in terms of the semispan; that is, the chord distribution is described as the curve of $\frac{c}{b/2}$ against $\frac{y}{b/2}$.

The span load distribution is similarly defined as the curve of $c_l \frac{c}{b/2}$ against $\frac{y}{b/2}$ and the induced-angle distribution is the curve of α_i against $\frac{y}{b/2}\left(-1 < \frac{y}{b/2} < 1\right)$.

The component loads are spanwise distributions of load intensity whose span lengths may be equal to or less than the wing span. They are added together to form the span load distributions. Their characteristics are described in this report by the following symbols:

E, value of $c_l \frac{c}{b/2}$ at the span center line of the initial elliptical component load that falls on the wing span center line.

δ, value of $c_l \frac{c}{b/2}$ at the span center line of an additional component load.

D, half-width of a component load. For the initial elliptical component, $D = b/2$. The nondimensional form is $\frac{D}{b/2}$.

I, subscript to identify quantities associated with the initial elliptical component load.

1, 2, ...n, subscripts to identify quantities associated with the first, the second, and the nth series of additional component loads, respectively.

y_n, spanwise distance of the center line of an nth component load from the span center line of the wing.

y_Δ, spanwise distance from the center line of a component load.

$\bar{\alpha}_a$, estimated weighted average of the given α_a distribution.

$\bar{a}_o$, estimated weighted average of the given a_o distribution.

A, wing aspect ratio.

c_t, wing-tip chord.

c_s, wing chord at span center line.

THEORY

The method is based on the principle of superposition: If individual load distributions are combined, their separate induced-angle distributions may be likewise added to give the distribution of induced angle associated with the resulting loading. Obviously, any desired span load distribution can be constructed by suitably combining a sufficient number of component loads. Essentially then, the method reduces to the process of determining the component loads which, when combined, will give the resultant load distribution and its associated induced-angle distribution that will satisfy the given problem.

Curve shapes suitable for processes of superposition must first be chosen. These shapes will constitute the "building blocks" of the method, the component loads. For convenience, the component loads and their associated distributions of induced angle should be easily defined and easily manipulated curves. The elliptical load distribution forms a satisfactory initial component (fig. 1), its span dimension being taken equal to the wing-span dimension and its associated induced angle being a constant value over the wing span. The additional component loads likely to be applied at any spanwise position should preferably not be elliptical in shape because the upwash velocity immediately beyond their tips would then be theoretically infinite in value. The curve shown in figure 2 avoids this difficulty and has been chosen in this report to form the additional components. With the shapes of the component loads decided upon, the procedure for determining their magnitudes, proportions, and positions for any given problem remains to be explained.

In any given problem, the plan form of the wing is generally known. Also given are the spanwise distributions of the section lift-curve slope a_0 and the geometric angle of attack from zero lift α_a. The procedure to be described involves, as a first step, the determination of a chord distribution that satisfies an elliptical span load distribution (the initial component) and the given distributions of angle of attack and section lift-curve slope. The differences between this fictitious and the given chord distributions define the additional component loads to be superposed upon the initial component to produce a new span load distribution that more nearly satisfies the given problem than did the elliptical component. The subsequent steps follow a similar procedure; the successive series of additional components, however, become rapidly negligible.

The initial elliptical component of the span load distribution is set up so that its magnitude is judged to be of the same general order as that of the resultant distribution sought. The distinguishing dimension of this component load is its ordinate at the center line E. A simple expression for estimating E will be given later. The value of E determines the magnitude of the corresponding induced angle α_{i_I}, which, of course, will be a constant over the span.

The subscript I is used to distinguish the quantities associated with the initial elliptical component load, and the following steps are defined:

At any spanwise point:

$$\alpha_{o_I} = \alpha_a - \alpha_{i_I}$$

$$c_{l_I} = a_o \; \alpha_{o_I}$$

$$\frac{c_I}{b/2} = \frac{\left(c_l \dfrac{c}{b/2} \right)_I}{c_{l_I}}$$

A chord distribution, $\frac{c_I}{b/2}$ against $\frac{y}{b/2}$, is thus obtained that satisfies the elliptical component load and the given α_a and a_o distributions. This chord distribution differs from the given chord distribution in a manner related to the differences between the elliptical component load and the actual load distribution sought. Obviously, if the two chord distributions should happen to agree, the elliptical load component would be the correct load distribution for the problem. These chord differences are now investigated:

$$\Delta \frac{c_I}{b/2} = \frac{c}{b/2} - \frac{c_I}{b/2}$$

The distributions of $\Delta \frac{c_I}{b/2}$ indicate where and how the initial load component should be modified to approach the resultant span load distribution being sought.

For each $\Delta \frac{c_I}{b/2}$ distribution, the following quantities are roughly noted: its width, taken as that portion of the span over which it maintains continuously positive or negative values; and its span position, taken as the location of the midpoint of its width. The next step is to apply an additional component load (the first additional loads) at each position of the $\Delta \frac{c_I}{b/2}$ distributions of a magnitude calculated to remove these chord differences.

The characteristic dimensions of each additional component load to be employed (fig. 2) are its ordinate at its midpoint, δ_1, and its half-breadth, $\frac{D_1}{b/2}$, where the subscript 1 is used to identify quantities corresponding to the first series of additional loads. The distribution of induced angle, $\Delta\alpha_{i_1}$, associated with an additional component, is defined by the values of δ_1 and $\frac{D_1}{b/2}$. (See fig. 2.) The quantity $\frac{D_1}{b/2}$ is set equal to the half-width of the $\Delta\frac{c_I}{b/2}$ distribution; and the value of δ_1 is so chosen that, at the position applied, the resulting load intensity $\left(c_l \frac{c}{b/2}\right)_1$ divided by the given a_0, and by the given α_a minus the resulting total induced angle α_{i_1}, will equal the given $\frac{c}{b/2}$. The induced interference at this span position of the other simultaneously applied additional component loads is generally small and is neglected in this calculation. A simple expression for determining δ_1 will be given later.

When the δ_1 loads just determined are thus added to the initial component load and the corresponding distributions of induced angle are similarly added, a new span distribution of load $\left(c_l \frac{c}{b/2}\right)_1$ and its associated induced-angle distribution α_{i_1} are obtained that approach the actual distributions sought. The procedure continues:

At any spanwise point;

$$\alpha_{o_1} = \alpha_a - \alpha_{i_1}$$

where $\alpha_{i_1} = \alpha_{i_I} + \Sigma\,\Delta\alpha_{i_1}$, $\Delta\alpha_{i_1}$ being the increment of induced angle corresponding to each δ_1 load,

$$c_{l_1} = a_o\,\alpha_{o_1}$$

$$\frac{c_1}{b/2} = \frac{\left(c_l \frac{c}{b/2}\right)_1}{c_{l_1}}$$

where $\left(c_l \dfrac{c}{b/2}\right)_1 = \left(c_l \dfrac{c}{b/2}\right)_I + \Delta\left(c_l \dfrac{c}{b/2}\right)_1$, $\Delta\left(c_l \dfrac{c}{b/2}\right)_1$ being the load increment associated with a δ_1 load.

A new chord distribution, $\dfrac{c_1}{b/2}$ against $\dfrac{y}{b/2}$, is thus obtained that satisfies the new span loading and the given α_a and a_o distributions. This $\dfrac{c_1}{b/2}$ distribution approaches the given $\dfrac{c}{b/2}$ distribution more closely than did the $\dfrac{c_I}{b/2}$ distribution.

The new chord differences are investigated:

$$\Delta \frac{c_1}{b/2} = \frac{c}{b/2} - \frac{c_1}{b/2}$$

A second series of additional component loads, the δ_2 loads, and their corresponding distributions of induced angle are obtained as before and added to the

$\left(c_l \dfrac{c}{b/2}\right)_1$ and α_{i_1} distributions to form the $\left(c_l \dfrac{c}{b/2}\right)_2$ and α_{i_2} distributions.

As before:

$$\alpha_{o_2} = \alpha_a - \alpha_{i_2}$$

$$c_{l_2} = a_o\ \alpha_{o_2}$$

$$\frac{c_2}{b/2} = \frac{\left(c_l \dfrac{c}{b/2}\right)_2}{c_{l_2}}$$

etc.

The repeated cycle of operations described is continued until the nth series of component loads obtained are negligible. The $\left(c_l \dfrac{c}{b/2}\right)_n$ distribution is then the distribution sought. Ordinarily, only three cycles are required.

APPLICATION

Formulas and Charts

<u>The initial component</u>.- Figure 1 is employed to set up the initial elliptical component load, the component width being the wing span. The center-line ordinate E may be obtained by the expression:

$$E = \frac{\overline{\alpha}_a}{7.16 + \frac{\pi A}{8\overline{a}_o}}$$

where $\overline{\alpha}_a$ and $\overline{a}_o$ are roughly estimated weighted averages of the α_a and a_o distributions, and A is the wing aspect ratio. The ordinates are scaled proportionately to E from figure 1. The induced-angle distribution associated with this load is a curve of constant ordinate:

$$\alpha_{i_I} = 7.16\,E \text{ degrees}$$

<u>The additional components</u>.- Figure 2 shows the curve shape chosen in this report to form the additional component loads. The characteristic dimensions of an additional component load are its half-breadth $\frac{D}{b/2}$ and its ordinate at the center line δ. The half-breadth $\frac{D}{b/2}$ is set equal to the half-breadth of the distribution-of-chord differences being considered, as explained in the previous section. The subscript 1 is appended to identify quantities associated with the first additional component loads; the subscript I identifies quantities associated with the initial elliptical component; and

$$\delta_1 = \frac{\left(a_o \frac{c}{b/2}\right)\alpha_{o_I} - \left(c_l \frac{c}{b/2}\right)_I}{1 + \left(a_o \frac{c}{b/2}\right)\left(\frac{12.16}{\frac{D_1}{b/2}}\right)}$$

where $\alpha_{o_I} = \alpha_a - \alpha_{i_I}$ at the spanwise position of the δ_1 load, $\left(c_l \frac{c}{b/2}\right)_I$ is the ordinate of the elliptical component load at that position, $\frac{D_1}{b/2}$ is the half-breadth of the δ_1 load, and the other quantities are given in the problem.

Similarly, the second additional component loads may subsequently be found:

$$\delta_2 = \frac{\left(a_o \frac{c}{b/2}\right)\alpha_{o_1} - \left(c_l \frac{c}{b/2}\right)_1}{1 + \left(a_o \frac{c}{b/2}\right)\left(\frac{12.16}{\frac{D_2}{b/2}}\right)}$$

and the nth additional components:

$$\delta_n = \frac{\left(a_o \frac{c}{b/2}\right)\alpha_{o_{n-1}} - \left(c_l \frac{c}{b/2}\right)_{n-1}}{1 + \left(a_o \frac{c}{b/2}\right)\left(\frac{12.16}{\frac{D_n}{b/2}}\right)}$$

<u>Charts for the additional components</u>.- In order to obtain the distribution of load associated with any additional component to be added to the curve of span load distribution, the load shape of figure 2 is expanded, the abscissas proportionately to the required value of $\frac{D}{b/2}$ and the ordinates proportionately to the required δ. The associated additional distribution of induced angle is similarly obtained by expanding the abscissas of the induced-angle curve of figure 2 proportionately to $\frac{D}{b/2}$ and the ordinates proportionately to $\frac{\delta}{\frac{D}{b/2}}$. The charts of figures 3 and 4 have been prepared to facilitate this procedure. These charts permit a rapid determi-

nation of the values of $\Delta\left(c_l \frac{c}{b/2}\right)$ and $\Delta\alpha_i$ to be applied at any span position, $\frac{y}{b/2}$ on the wing for any given additional load position $\left(\frac{y}{b/2} = \frac{y_n}{b/2}\right)$ and for any values of $\frac{D}{b/2}$ and δ within the range of usefulness. The simple construction of the charts readily allows them to be reproduced to a larger scale than is possible for publication.

The chart of figure 3 is used as follows: The $\frac{y}{b/2}$ scale, which represents the wing span, is shifted to bring the δ-load center line to its desired position, $\frac{y_n}{b/2}$. A horizontal base line is established in the fan of y_Δ lines so that its length from $y_\Delta = 0$ to $y_\Delta = \pm 1.0\ D$ is equal to the given $\frac{D_n}{b/2}$. The point on the span under consideration is projected vertically from the shifted $\frac{y}{b/2}$ scale to this base line in the y_Δ fan and is then carried radially to the δ-load base line. It is then projected vertically to intersect a curve that is picked or interpolated from the family of δ-load curves shown. This curve is so chosen that it has the given value of δ_n as indicated on one of the $\Delta\left(c_l \frac{c}{b/2}\right)$ scales. The ordinate of the intersection as read on the same $\Delta\left(c_l \frac{c}{b/2}\right)$ scale is the value of $\Delta\left(c_l \frac{c}{b/2}\right)_n$ to be added to the existing span load distribution at the spanwise point being considered and corresponds to the given values of $\frac{y_n}{b/2}$, $\frac{D_n}{b/2}$, and δ_n. Note that any of the δ-load curves can be used with any of the scales to give the desired value of δ_n. Further, any value of the exponent x associated with the scale can be used to fix the decimal point as required. The procedure just described is illustrated by broken lines in figure 3 for $\frac{y_n}{b/2}$, $\frac{y}{b/2}$, $\frac{D_n}{b/2}$, and δ_n assumed equal to 0.35, 0.45, 0.25, and 0.15, respectively. The corresponding value of $\Delta\left(c_l \frac{c}{b/2}\right)$ is 0.126.

Figure 4 is used similarly to figure 3. The induced-angle curve to be employed, however, is picked or interpolated, from the family of curves shown, to have the desired value of $\dfrac{\frac{\delta}{D}}{b/2}$. In the process of obtaining the distribution of $\Delta\alpha_i$ corresponding to a given δ-load, the distribution is considered to extend sufficiently far over the span to reach negligible values of $\Delta\alpha_i$. The procedure of using the chart is illustrated in figure 4 for the same values of $\frac{y_n}{b/2}$, $\frac{y}{b/2}$, $\frac{D_n}{b/2}$, and δ_n assumed in figure 3. The corresponding $\Delta\alpha_{i_n}$ is 4.8°.

<u>Example</u>.- An example is presented in detail to illustrate the application of the method. Figure 5 and table I give the data assumed to be known.

Eperience has shown that it is much more convenient to perform the computations not covered by the charts in tabular form rather than by graphical construction. Table II presents the work necessary for deriving the load distributions. Table III gives the details of deriving the distributions of induced angle involved in getting the chord differences of table II. Figure 6 is included to illustrate the results of the tabular work. The check points shown were obtained by graphical integration, as described in reference 7. In this example, the third additional component loads are considered negligible and the $\left(c_l \frac{c}{b/2}\right)_a$ distribution is accepted as satisfactory. Following through the example is considered advisable to obtain a working understanding of the method.

The method as presented in this report may accumulate inaccuracies during computation. In simple examples, as the one given herein, this source of error is not serious. In very complex problems, where many series of additional loads are required, the accumulation of computing inaccuracies might become important. At an nth stage, it would therefore be advisable to derive the induced-angle distribution associated with the curve of $\left(c_l \frac{c}{b/2}\right)_n$ also by graphical means (see reference 7) or by means of an integrator such as described in reference 8 to check the α_{i_n} curve. This proceeding removes the error accumulated and the method can be continued afresh from that point.

In the working of an example, the induced loads produced by the upwash at the tips of an additional component must not be neglected. In tabular procedure, it is sufficient to make each component-load tip fall on a value of $\frac{y}{b/2}$ considered in the table. With the point of application, $\frac{y_n}{b/2}$, of the additional load also falling on such a value, the tabular work is less than it otherwise would be because the values of $\Delta\left(c_l \frac{c}{b/2}\right)_n$ and $\Delta\alpha_{i_n}$ to be tabulated are symmetrical about $\frac{y_n}{b/2}$. (See tables II and III.)

CONCLUSION

The method presented should enable the engineer to obtain the span load distribution through a straightforward, arithmetical procedure in which component loads are added to form the distribution sought. The proportions, the magnitudes, and the relative span positions of these component loads are determined from simple relationships derived from the differences between the given chord distribution and the chord distributions associated with the components themselves. Charts have been included to assist in the calculations.

Points of procedure to bear in mind are:

1. Tabular computation is more convenient than graphical construction.

2. The interference loads induced at the tips of each additional component load must not be overlooked, as is possible in tabular computation.

3. The accumulation of computing inaccuracies, though generally unimportant in simple problems, may become serious in problems requiring the addition of many series of

component loads. At any stage in the process, the accumulation of inaccuracies can be removed by a separate check of the associated induce-angle distribution.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., August 16, 1939.

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TABLE I

Characteristics of Wing Used in Example

$\frac{y}{b/2}$	$\frac{c}{b/2}$	a_o	α_a (deg.)		$a_o \frac{c}{b/2}$	$\bar{a}_o$	$\bar{\alpha}_a$ (deg.)
-1.00	(0.075)	0.100	7.0		(0.0075)	0.096	8.7
-.95	.089	.100	7.1		.0089		
-.90	.103	.099	7.2		.0102		
-.85	.117	.099	7.4		.0116		
-.80	.131	.099	7.5		.0130		
-.75	.145	.098	7.6		.0142		
-.70	.159	.098	7.7		.0156		
-.65	.173	.098	7.8		.0170		
-.60	.187	.097	8.0		.0181		
-.55	.201	.097	8.1		.0195		
-.50	.215	.097	8.2		.0209		
-.45	.229	.096	8.3		.0220		
-.40	.243	.096	8.5		.0233		
-.35	.257	.096	8.6		.0247		
-.30	.272	.095	8.7		.0258		
-.25	.286	.095	8.9		.0272		
-.20	.300	.095	9.0		.0285		
-.15	.300	.095	9.0		.0285		
-.10	.300	.095	9.0		.0285		
-.05	.300	.095	9.0		.0285		
0	.300	.095	9.0		.0285		
.05	.300	.095	9.0		.0285		
.10	.300	.095	9.0		.0285		
.15	.300	.095	9.0		.0285		
.20	.300	.095	9.0		.0285		
.25	.286	.095	8.9		.0272		
.30	.272	.095	8.7		.0258		
.35	.257	.096	8.6		.0247		
.40	.243	.096	8.5		.0233		
.45	.229	.096	8.3		.0220		
.50	.215	.097	8.2		.0209		
.55	.201	.097	8.1		.0195		
.60	.187	.097	8.0		.0181		
.65	.173	.098	7.8		.0170		
.70	.159	.098	7.7		.0156		
.75	.145	.098	7.6		.0142		
.80	.131	.099	7.5		.0130		
.85	.117	.099	7.4		.0116		
.90	.103	.099	7.2		.0102		
.95	.089	.100	7.1		.0089		
1.00	(.075)	.100	7.0		(.0075)		

TABLE II

Computations for example. Spanwise distributions.

For initial elliptical component of span-loading curve								For first additional components									
$\frac{y}{b/2}$	E	$\left(c_l \frac{c}{b/2}\right)_I$	α_{l_I}	α_{o_I}	c_{l_I}	$\frac{c_I}{b/2}$	$\Delta\frac{c_I}{b/2}$	$\frac{y_1}{b/2}$	$\frac{D_1}{b/2}$	δ_1	$\Delta\left(c_l \frac{c}{b/2}\right)_1$	$\left(c_l \frac{c}{b/2}\right)_1$	α_{l_1}	α_{o_1}	c_{l_1}	$\frac{c_1}{b/2}$	$\Delta\frac{c_1}{b/2}$
-1.00		0	1.4	5.6	0.560	0	—					0	1.6	5.4	0.540	0	—
-.95		.058	1.4	5.7	.570	.102	-0.013				0	.058	1.8	5.3	.530	.109	-0.020
-.90		.082	1.4	5.8	.574	.143	-.040				-.002	.080	1.8	5.4	.535	.149	-.046
-.85		.099	1.4	6.0	.594	.167	-.050				-.006	.093	1.5	5.9	.584	.159	-.042
-.80		.113	1.4	6.1	.604	.187	-.056				-.011	.102	1.1	6.4	.634	.161	-.030
-.75		.124	1.4	6.2	.608	.204	-.059				-.016	.108	.9	6.7	.657	.164	-.019
-.70		.134	1.4	6.3	.617	.217	-.058				-.019	.115	.7	7.0	.686	.168	-.009
-.65		.143	1.4	6.4	.627	.228	-.055	-0.65	0.30	-0.020	-.020	.123	.6	7.2	.706	.174	-.001
-.60		.151	1.4	6.6	.640	.236	-.049				-.019	.132	.7	7.3	.708	.186	.001
-.55		.157	1.4	6.7	.650	.242	-.041				-.016	.141	.9	7.2	.699	.202	-.001
-.50		.163	1.4	6.8	.660	.247	-.032				-.011	.152	1.0	7.2	.699	.217	-.002
-.45		.168	1.4	6.9	.663	.253	-.024				-.006	.162	1.4	6.9	.663	.244	-.015
-.40		.173	1.4	7.1	.682	.254	-.011				-.002	.171	1.7	6.8	.653	.262	-.019
-.35		.176	1.4	7.2	.691	.255	.002				0	.176	1.6	7.0	.672	.262	-.005
-.30		.179	1.4	7.3	.694	.258	.014				.001	.180	1.3	7.4	.703	.256	.016
-.25		.182	1.4	7.5	.713	.255	.031				.003	.185	1.4	7.5	.713	.260	.026
-.20		.184	1.4	7.6	.722	.255	.045				.006	.190	1.6	7.4	.703	.270	.030
-.15		.186	1.4	7.6	.722	.258	.042				.009	.195	1.7	7.3	.694	.281	.019
-.10		.187	1.4	7.6	.722	.259	.041				.012	.199	1.9	7.1	.673	.296	.004
-.05		.188	1.4	7.6	.722	.260	.040				.013	.201	1.9	7.1	.673	.299	.001
0	0.189	.189	1.4	7.6	.722	.262	.038	0	.35	.014	.014	.203	1.9	7.1	.673	.302	-.002
.05		.188	1.4	7.6	.722	.260	.040				.013	.201	1.9	7.1	.673	.299	.001
.10		.187	1.4	7.6	.722	.259	.041				.012	.199	1.9	7.1	.673	.296	.004
.15		.186	1.4	7.6	.722	.258	.042				.009	.195	1.7	7.3	.694	.281	.019
.20		.184	1.4	7.6	.722	.255	.045				.006	.190	1.6	7.4	.703	.270	.030
.25		.182	1.4	7.5	.713	.255	.031				.003	.185	1.4	7.5	.713	.260	.026
.30		.179	1.4	7.3	.694	.258	.014				.001	.180	1.3	7.4	.703	.256	.016
.35		.176	1.4	7.2	.691	.255	.002				0	.176	1.6	7.0	.672	.262	-.005
.40		.173	1.4	7.1	.682	.254	-.011				-.002	.171	1.7	6.8	.653	.262	-.019
.45		.168	1.4	6.9	.663	.253	-.024				-.006	.162	1.4	6.9	.663	.244	-.015
.50		.163	1.4	6.8	.660	.247	-.032				-.011	.152	1.0	7.2	.699	.217	-.002
.55		.157	1.4	6.7	.650	.242	-.041				-.016	.141	.9	7.2	.699	.202	-.001
.60		.151	1.4	6.6	.640	.236	-.049				-.019	.132	.7	7.3	.708	.186	.001
.65		.143	1.4	6.4	.627	.228	-.055	.65	.30	-.020	-.020	.123	.6	7.2	.706	.174	-.001
.70		.134	1.4	6.3	.617	.217	-.058				-.019	.115	.7	7.0	.686	.168	-.009
.75		.124	1.4	6.2	.608	.204	-.059				-.016	.108	.9	6.7	.657	.164	-.019
.80		.113	1.4	6.1	.604	.187	-.056				-.011	.102	1.1	6.4	.634	.161	-.030
.85		.099	1.4	6.0	.594	.167	-.050				-.006	.093	1.5	5.9	.584	.159	-.042
.90		.082	1.4	5.8	.574	.143	-.040				-.002	.080	1.8	5.4	.535	.149	-.046
.95		.058	1.4	5.7	.570	.102	-.013				0	.058	1.8	5.3	.530	.109	-.020
1.00		0	1.4	5.6	.560	0	—					0	1.6	5.4	.540	0	—

TABLE II (continued)

For second additional components											For residual components		
$\frac{y}{b/2}$	$\frac{y_s}{b/2}$	$\frac{D_s}{b/2}$	δ_s	$\Delta\left(c_l\frac{c}{b/2}\right)_s$	$\left(c_l\frac{c}{b/2}\right)_s$	α_{i_s}	α_{o_s}	c_{l_s}	$\frac{c_s}{b/2}$	$\Delta\ \frac{c_s}{b/2}$	$\frac{y_s}{b/2}$	$\frac{D_s}{b/2}$	δ_s
-1.00				0	0	2.1	4.9	0.490	0	—			
-.95				-.004	.054	1.9	5.2	.520	.104	0.015			
-.90				-.010	.070	1.1	6.1	.604	.116	-.013	-0.90	0.10	-0.004
-.85	-.85	.15	-0.013	-.013	.080	.4	7.0	.693	.115	.002			
-.80				-.010	.092	.4	7.1	.703	.131	0			
-.75				-.004	.104	1.0	6.6	.647	.161	-.016			
-.70				0	.115	1.2	6.5	.637	.180	-.021	-.70	.10	-.005
-.65					.123	.8	7.0	.686	.179	-.006			
-.60					.132	.8	7.2	.699	.189	-.002			
-.55					.141	1.0	7.1	.689	.205	-.004			
-.50				0	.152	1.2	7.0	.679	.224	-.009	-.50	.10	-.002
-.45				-.002	.160	1.3	7.0	.672	.238	-.009			
-.40	-.40	.10	-.003	-.003	.168	1.2	7.3	.701	.240	.003			
-.35				-.002	.174	1.3	7.3	.701	.248	.009			
-.30				.002	.182	1.4	7.3	.694	.262	.010	-.30	.10	.001
-.25				.005	.190	1.7	7.2	.684	.278	.008			
-.20	-.20	.15	.006	.006	.196	2.1	6.9	.655	.299	.001			
-.15				.005	.200	2.0	7.0	.665	.301	-.001			
-.10				.002	.201	1.8	7.2	.684	.294	.006			
-.05				0	.201	1.7	7.3	.694	.290	.010			
0					.203	1.7	7.3	.694	.293	.007	0	.15	.002
.05				0	.201	1.7	7.3	.694	.290	.010			
.10				.002	.201	1.8	7.2	.684	.294	.006			
.15				.005	.200	2.0	7.0	.665	.301	-.001			
.20	.20	.15	.006	.006	.196	2.1	6.9	.655	.299	.001			
.25				.005	.190	1.7	7.2	.684	.278	.008			
.30				.002	.182	1.4	7.3	.694	.262	.010	.30	.10	.001
.35				-.002	.174	1.3	7.3	.701	.248	.009			
.40	.40	.10	-.003	-.003	.168	1.2	7.3	.701	.240	.003			
.45				-.002	.160	1.3	7.0	.672	.238	-.009			
.50				0	.152	1.2	7.0	.679	.224	-.009	.50	.10	-.002
.55					.141	1.0	7.1	.689	.205	-.004			
.60					.132	.8	7.2	.699	.189	-.002			
.65					.123	.8	7.0	.686	.179	-.006			
.70				0	.115	1.2	6.5	.637	.180	-.021	.70	.10	-.005
.75				-.004	.104	1.0	6.6	.647	.161	-.016			
.80				-.010	.092	.4	7.1	.703	.131	0			
.85	.85	.15	-.013	-.013	.080	.4	7.0	.693	.115	.002			
.90				-.010	.070	1.1	6.1	.604	.116	-.013	.90	.10	-.004
.95				-.004	.054	1.9	5.2	.520	.104	-.015			
1.00				0	0	2.1	4.9	.490	0	—			

TABLE III

Derivations of α_i distributions from chart of $\Delta\alpha_i$ (fig. 4)

	For first additional components								For second additional components									
$\frac{y}{b/2}$	$\frac{D_1}{b/2}$	$\frac{\delta_1}{D_1/(b/2)}$	$\Delta\alpha_i$ (deg.)	$\Delta\alpha_i$ (deg.)	$\Delta\alpha_i$ (deg.)	$\Sigma\Delta\alpha_i$ (deg.)	α_{i_I} (deg.)	α_{i_1} (deg.)	$\frac{D_2}{b/2}$	$\frac{\delta_2}{D_2/(b/2)}$	$\Delta\alpha_{i_2}$ (deg.)	$\Delta\alpha_{i_2}$ (deg.)	$\Delta\alpha_{i_2}$ (deg.)	$\Delta\alpha_{i_2}$ (deg.)	$\Delta\alpha_{i_2}$ (deg.)	$\Delta\alpha_{i_2}$ (deg.)	$\Sigma\Delta\alpha_{i_2}$ (deg.)	α_{i_2} (deg.)
-1.00			0.2			0.2	1.4	1.6			0.5						0.5	2.1
-.95			.4			.4	1.4	1.8			.1						.1	1.9
-.90			.4			.4	1.4	1.8			-.7						-.7	1.1
-.85			.1			.1	1.4	1.5	0.15	-0.087	-1.1						-1.1	.4
-.80			-.3			-.3	1.4	1.1			-.7						-.7	.4
-.75			-.5			-.5	1.4	.9			.1						.1	1.0
-.70			-.7			-.7	1.4	.7			.5						.5	1.2
-.65	0.30	-0.067	-.8			-.8	1.4	.6			.2						.2	.8
-.60			-.7			-.7	1.4	.7			.1						.1	.8
-.55			-.5	0		-.5	1.4	.9			.1	0					.1	1.0
-.50			-.3	-.1		-.4	1.4	1.0			0	.2					.2	1.2
-.45			.1	-.1		0	1.4	1.4				-.1	0				-.1	1.3
-.40			.4	-.1		.3	1.4	1.7	.10	-.030		-.4	-.1				-.5	1.2
-.35			.4	-.2		.2	1.4	1.6				-.1	-.2				-.3	1.3
-.30			.2	-.3		-.1	1.4	1.3				.2	-.1				.1	1.4
-.25			.1	-.1		0	1.4	1.4				0	.3				.3	1.7
-.20			.1	.1		.2	1.4	1.6	.15	.040			.5				.5	2.1
-.15			.1	.2		.3	1.4	1.7					.3				.3	2.0
-.10			.1	.4		.5	1.4	1.9					-.1				-.1	1.8
-.05			0	.5		.5	1.4	1.9					-.2	0			-.2	1.7
0	.35	.040		.5		.5	1.4	1.9					-.1	-.1			-.2	1.7
.05				.5	0	.5	1.4	1.9					0	-.2			-.2	1.7
.10				.4	.1	.5	1.4	1.9						-.1			-.1	1.8
.15				.2	.1	.3	1.4	1.7						.3			.3	2.0
.20				.1	.1	.2	1.4	1.6	.15	.040				.5			.5	2.1
.25				-.1	.1	0	1.4	1.4						.3	0		.3	1.7
.30				-.3	.2	-.1	1.4	1.3						-.1	.2		.1	1.4
.35				-.2	.4	.2	1.4	1.6						-.2	-.1		-.3	1.3
.40				-.1	.4	.3	1.4	1.7	.10	-.030				-.1	-.4		-.5	1.2
.45				-.1	.1	0	1.4	1.4						0	-.1		-.1	1.3
.50				-.1	-.3	-.4	1.4	1.0							.2	0	.2	1.2
.55				0	-.5	-.5	1.4	.9							0	.1	.1	1.0
.60					-.7	-.7	1.4	.7								.1	.1	.8
.65	.30	-.067			-.8	-.8	1.4	.6								.2	.2	.8
.70					-.7	-.7	1.4	.7								.5	.5	1.2
.75					-.5	-.5	1.4	.9								.1	.1	1.0
.80					-.3	-.3	1.4	1.1								-.7	-.7	.4
.85					.1	.1	1.4	1.5	.15	-.087						-1.1	-1.1	.4
.90					.4	.4	1.4	1.8								-.7	-.7	1.1
.95					.4	.4	1.4	1.8								.1	.1	1.9
1.00					.2	.2	1.4	1.6								.5	.5	2.1

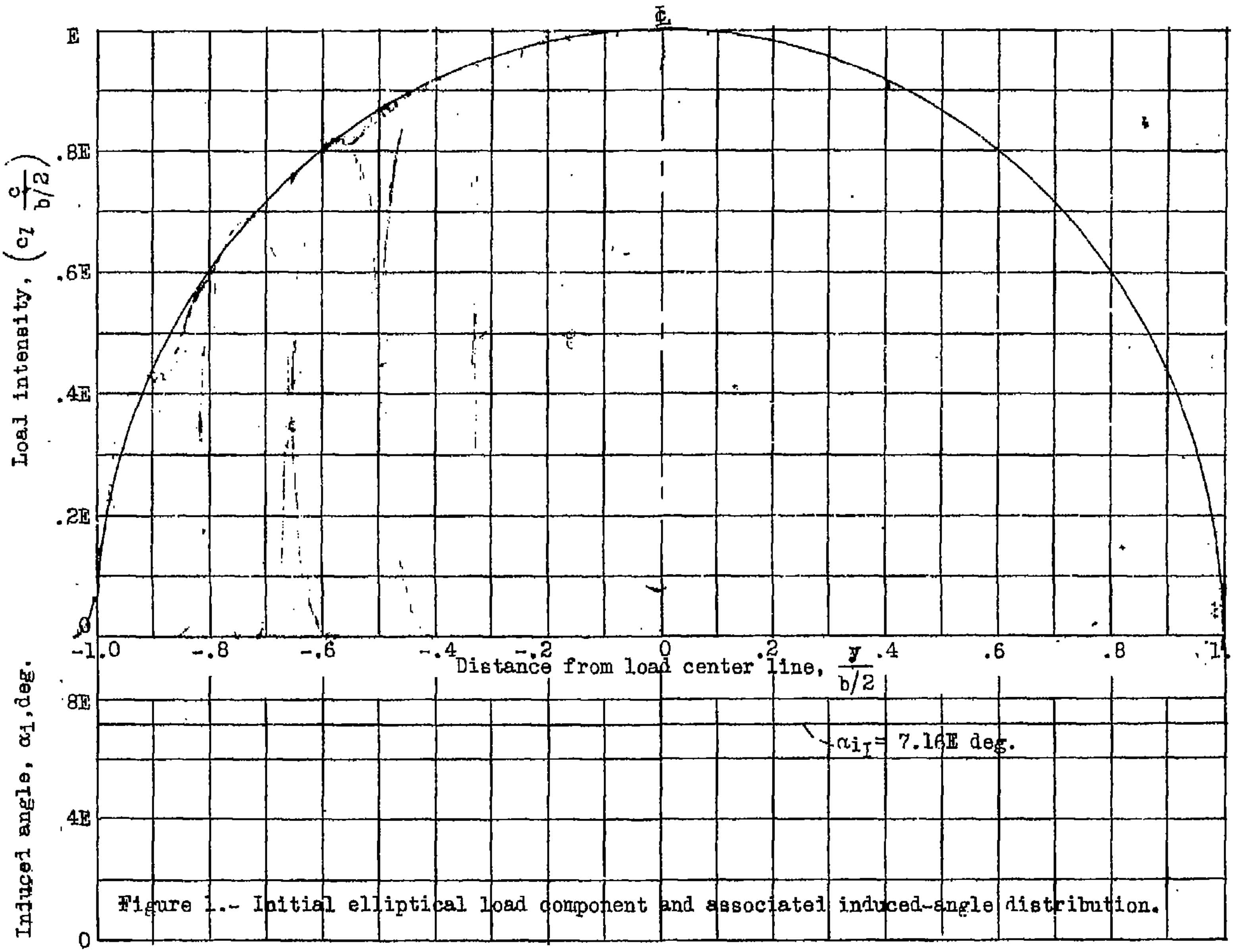

Figure 1.- Initial elliptical load component and associated induced-angle distribution.

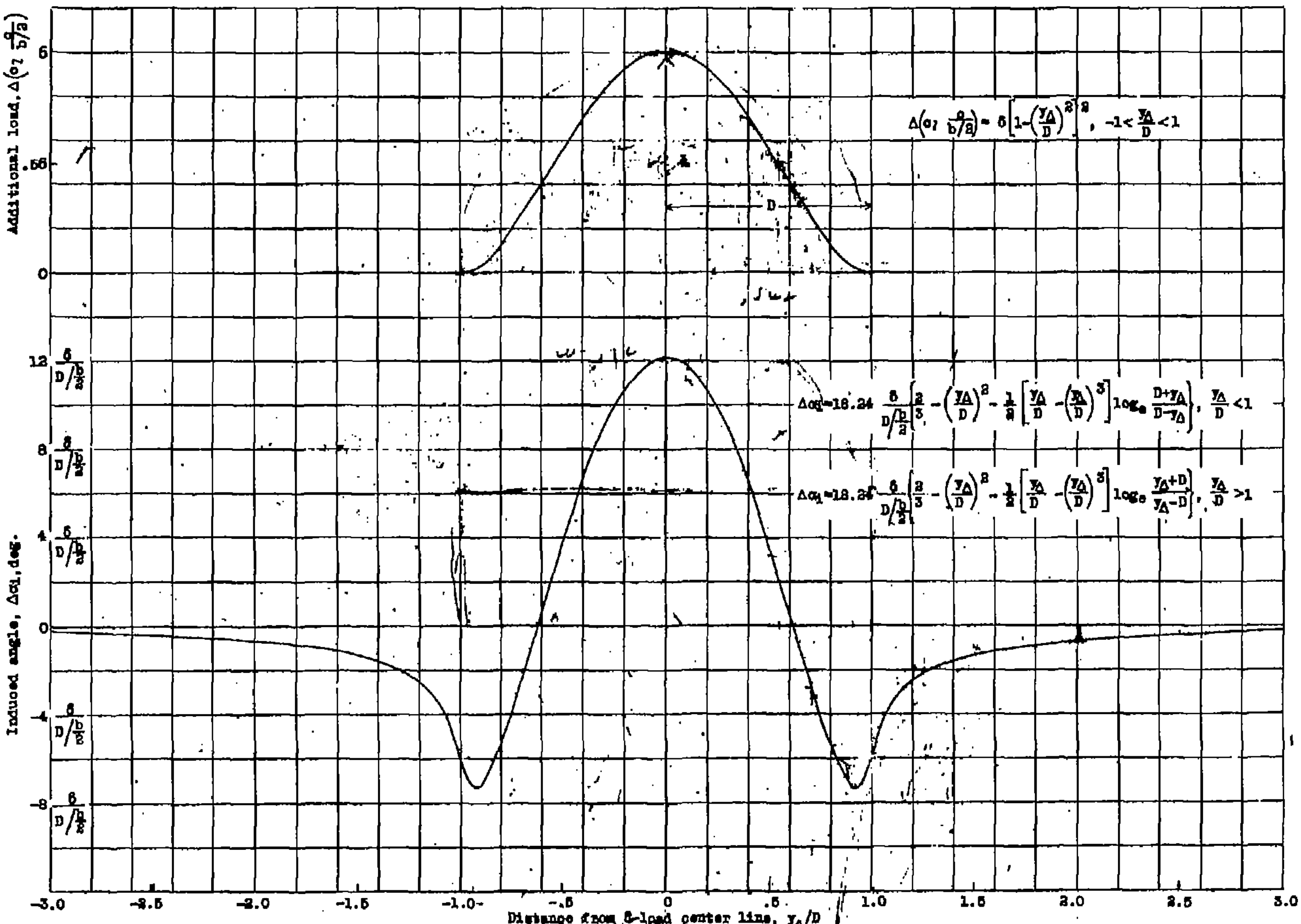

Figure 2.- Additional load and associated induced-angle distributions.

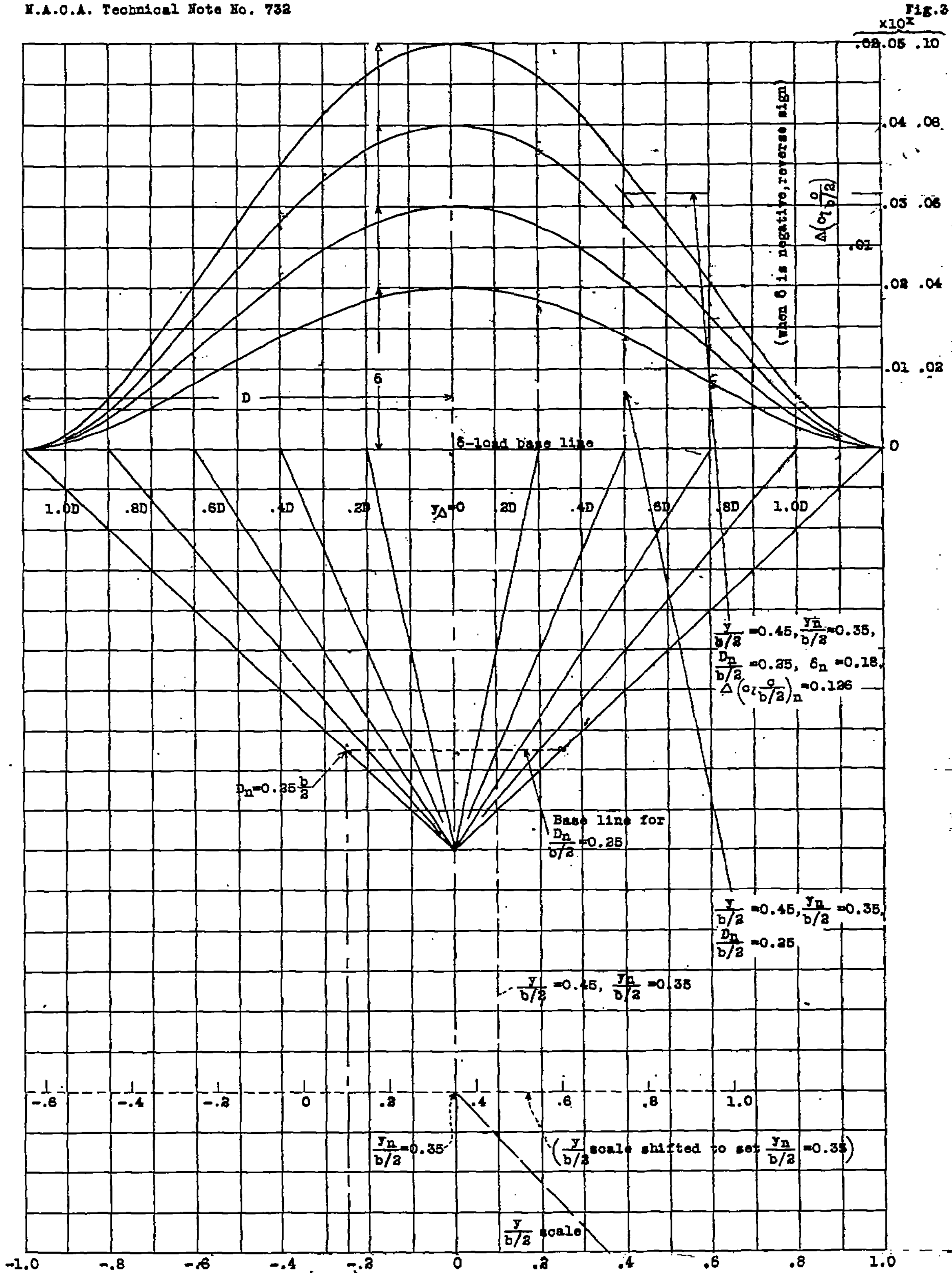

Figure 3.- Chart for obtaining $\Delta\left(c_l \frac{c}{b/2}\right)$ distributions. The broken lines illustrate the determination of the value of $\Delta\left(c_l \frac{c}{b/2}\right)_n$ corresponding to $\frac{y}{b/2}$, $\frac{y_n}{b/2}$, $\frac{D_n}{b/2}$, and δ_n taken equal to 0.45, 0.35, 0.25, and 0.18, respectively.

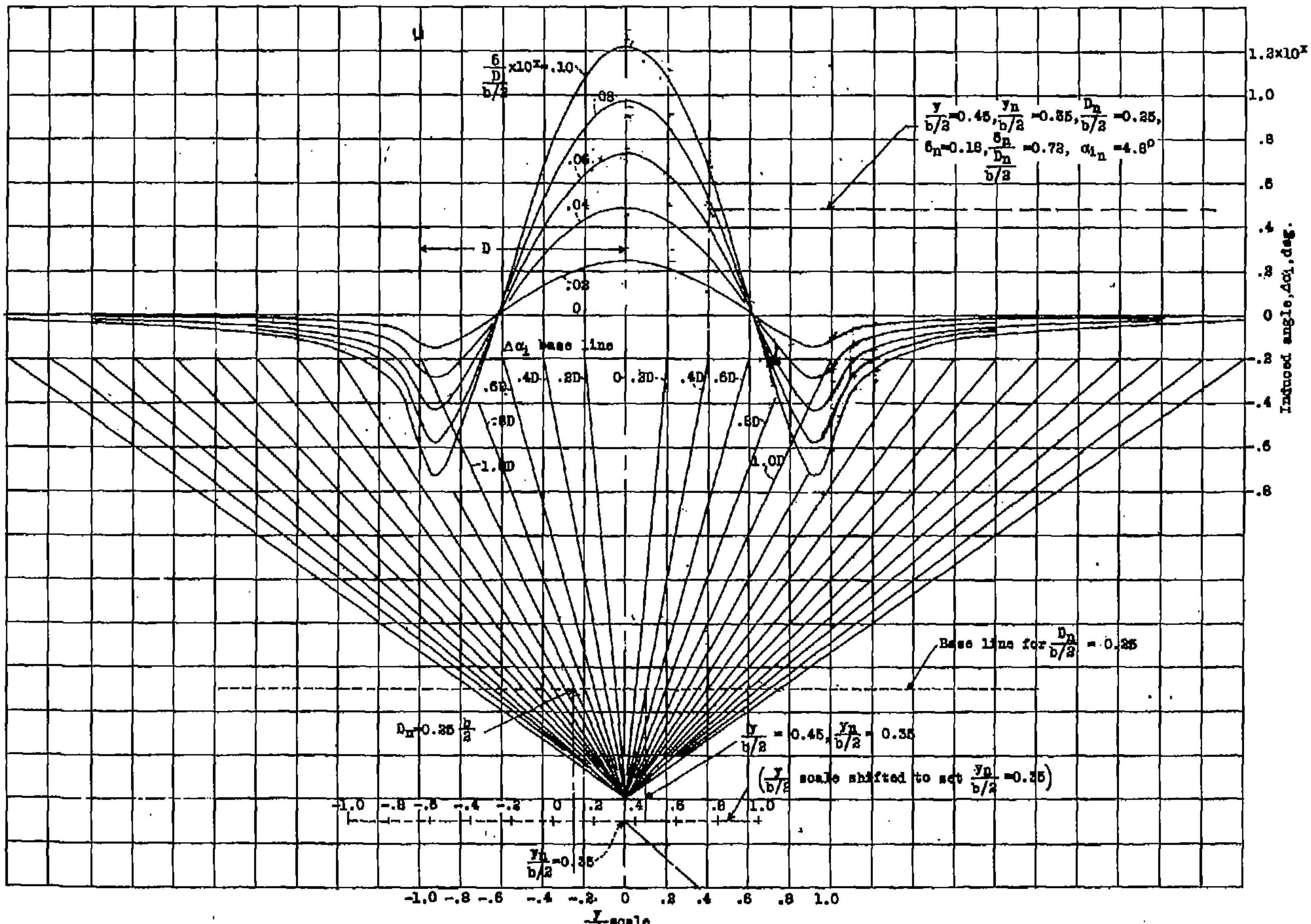

Figure 4.- Chart for obtaining $\Delta\alpha_i$ distributions. The broken lines illustrate the determination of the value of $\Delta\alpha_i$ corresponding to $\frac{y}{b/2}$, $\frac{y_n}{b/2}$, $\frac{D_n}{b/2}$, and δ_n taken equal to 0.45, 0.35, 0.25, and 0.18, respectively.

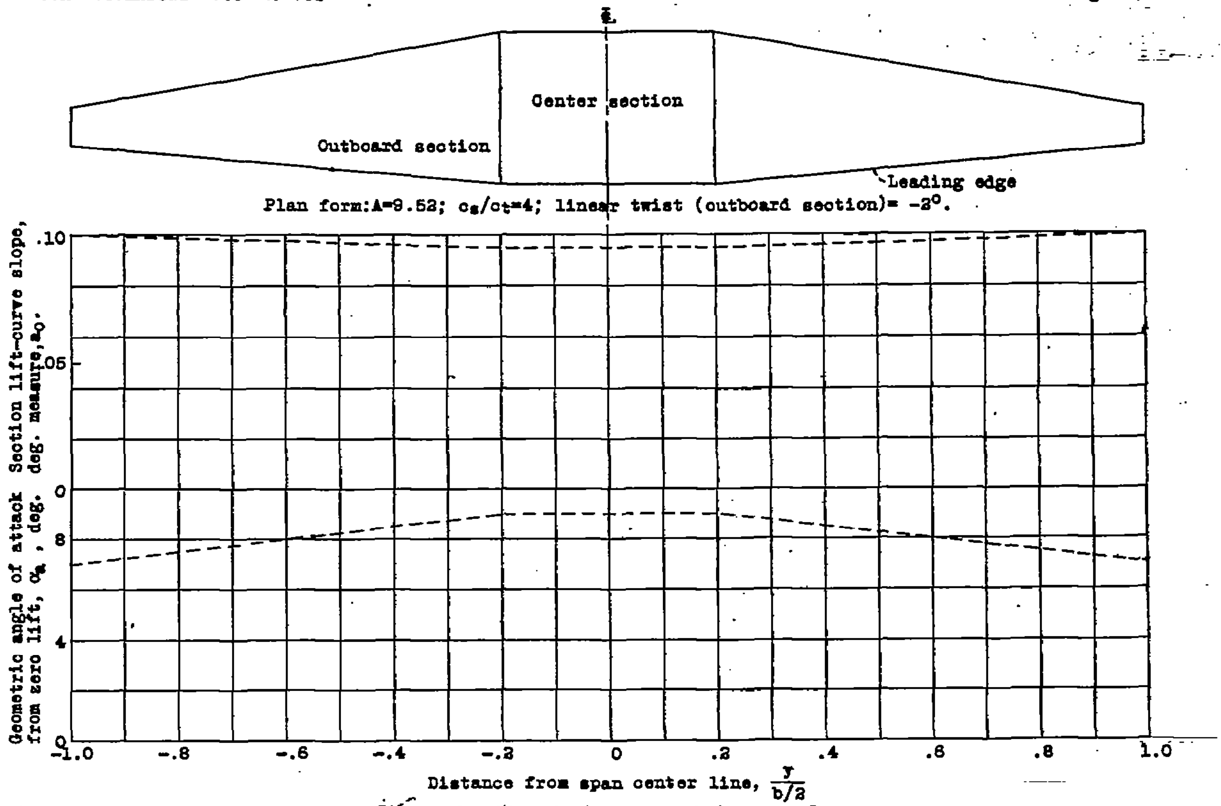

Figure 5.- Wing design assumed for example.

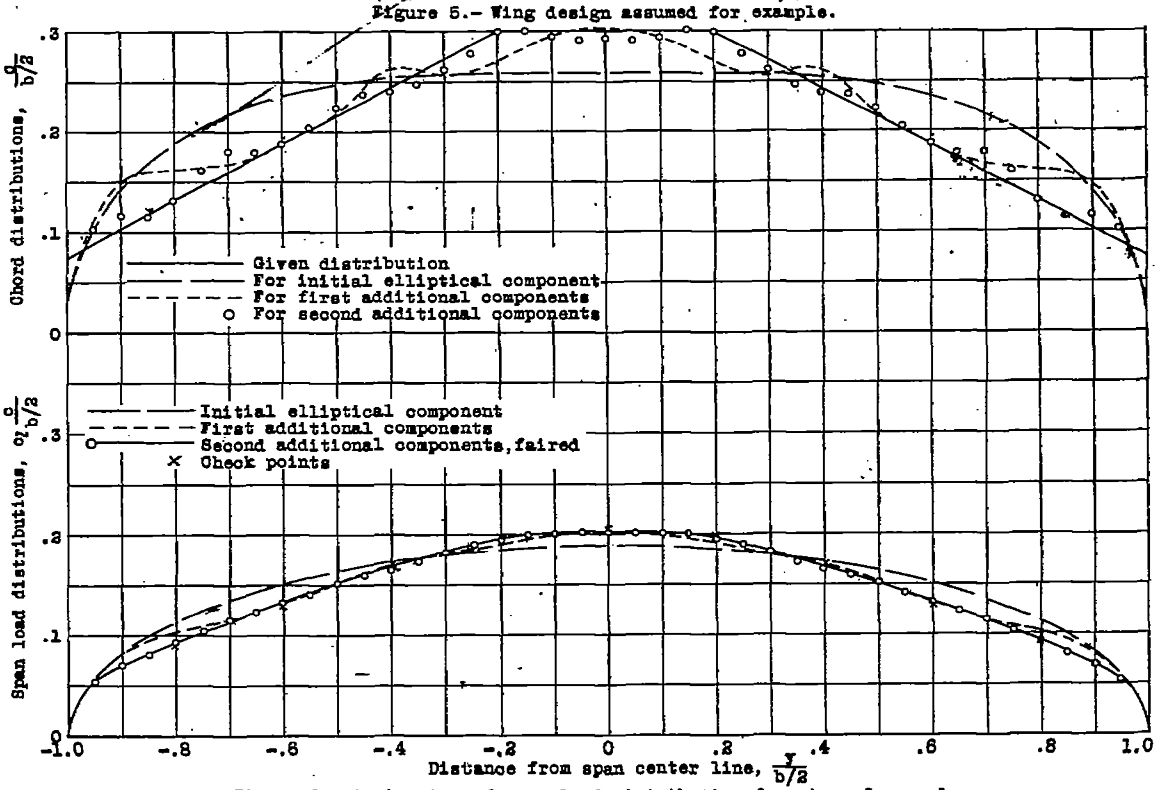

Figure 6.- Derivation of span load distribution for wing of example.